

Statistical Physics



Ludwig Boltzmann, 1844 - 1906

Information

- **lecture:**

- Tuesdays and Thursdays , 11:00-12:15

- **homework:**

- typically every other week
- solution discussed in the next lecture after due date
- posted online

- **webpage:**

http://www.aglatz.net/teaching/statphys_S2024/

- **office hours:**

Virtual by appointment, aglatz@niu.edu

Tue&Thu: 12:15-13:15

Exams & Grading

- **Exams (tentatively):**

- *Midterm:* March 19th, 2024, 11:00-12:15.

- *Final:* May 7, 2024.

- Closed book
- You may bring one page of notes in your own original handwriting
- Relevant formulas will be given with the problems
- No electronic devices are allowed

Final grade:

45%: homework percentage

10%: lecture attendance percentage

20%: midterm exam percentage

25%: final exam percentage

➔ total score between 0 and 1

➔ multiplied by 12

➔ rounded to the closed integer

➔ divided by 3, and finally graded according to ➔

You MUST score at least 50% on the homework.

A = 4.00
A- = 3.67
B+ = 3.33
B = 3.00
B- = 2.67
C+ = 2.33
C = 2.00
D = 1.00
F = 0.00

Role and tasks of Statistical Physics

For the description of physical systems we know the following “microscopic” theories:

(i) classical (deterministic) theories

- **mechanics:** Lagrangian/Hamiltonian – exact solutions limited to few degrees of freedom (DoF) : *three-body problem already non-solvable*; description and solution of kinetic equations for gases or the solid state is hopeless (10^{20} - 10^{23} DoF, for molecules $\sim 10^9$).
- **electrodynamics:** electromagnetic fields have “ ∞ ” DoF [more precisely $(L/\lambda_c)^3$], Compton wave length $\lambda_c = h/(mc) \approx 2.4 \times 10^{-10} \text{cm} \rightarrow 10^{31} \text{ DoF/cm}^3$

linear theory \rightarrow solution by superposition possible,
but with interaction to matter \rightarrow effective non-linear \rightarrow no general solution

Even if kinetic equations could be solved, the initial conditions are unknown
 \rightarrow deterministic chaos (strong dependence on initial conditions)

(ii) Quantum theories (quantum mechanics/electrodynamics)

in principle these are statistical theories: determine the probability to obtain a certain result in a measurement. Solution of the Schrödinger for 10^{20} interacting degrees of freedom is also hopeless.

Goal of statistical physics is to explain the macroscopic behavior (like pressure, specific heat, conductivity, etc.) of many-body systems, starting from their microscopic description.

“Heat”, “Temperature”, “thermal equilibrium”, etc. are properties and terms, which we can experience and observe, but they did not appear in microscopic theories so far.

At first it looks like a futile task to describe many ($\sim 10^{20}$) degrees of freedom, but it turns out this apparent disadvantage can be beneficial to describe the macroscopic behavior of a system:

For those, **one does not need to know all details** of the fast microscopic variables, but rather slow ones – especially **conserved quantities** – are most important.

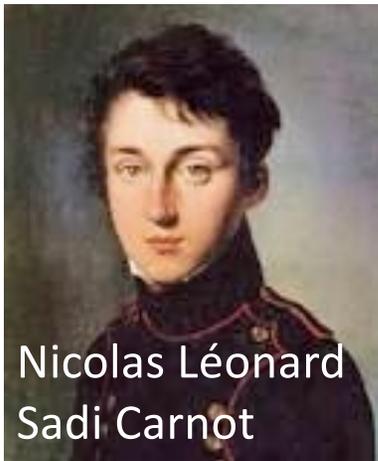
Areas of statistical physics

- i. **statistical (quantum) mechanics**, (quantum) electrodynamics
- ii. theory of disordered solids (in solids impurities are usually randomly distributed)
- iii. theory of neural networks, pre-biological evolution of molecules
- iv. quantum mechanics in path-integral formalism

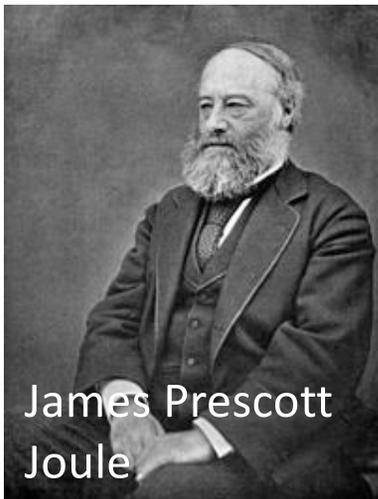
Short history



Anders Celsius

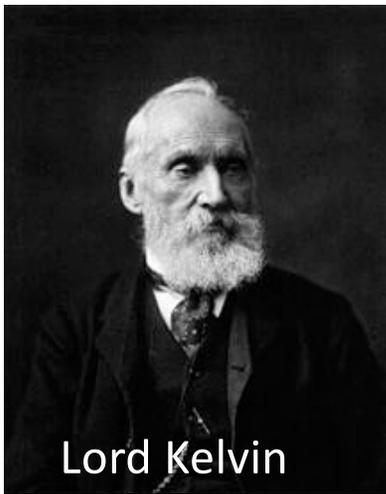


Nicolas Léonard
Sadi Carnot



James Prescott
Joule

- **17. century** (*Ferdinand II*): quantitative measurement of temperature (alcohol thermometer)
- **18. century** *Celsius*: temperature scale
- *Josef Black* (1728–1799): first measurements for heat equilibrium of connected bodies
- **1738** *Bernoulli*: pressure in fluids $p \sim \rho v^2$
- **1802** *Gay-Lussac* (1778–1850): heat expansion of gases (latent heat, heat as elastic fluid)
- *Benjamin Thompson (Earl Rumford)* (1753–1814): equivalence of mechanical and heat energy
- **1822** *J.B.J. Fourier* (1768–1830): equation for heat conduction
- **1824** *N.L.S. Carnot*: description of heat engines, Carnot cycle
- **1841** *J.P. Joule*: $Q \sim I^2 R t$ (heat generated by electric currents)
- **1842–45** *J.R. Mayer*: equivalence of heat and work, energy conservation
- **1847** *H.v. Helmholtz*: modern form of the energy conservation law (first law of thermodynamics)
- **1848** *W. Thomson (Lord Kelvin)*: definition of the thermodynamic temperature scale (using the Carnot process)



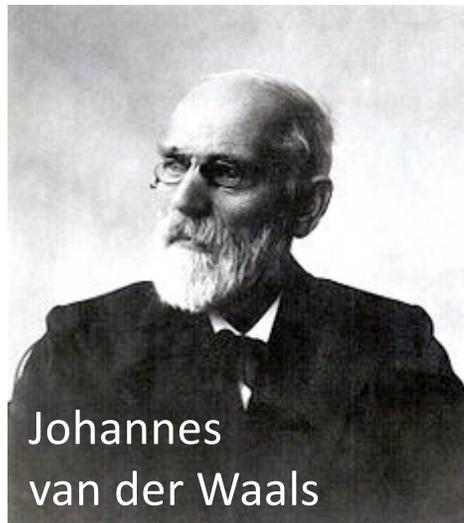
Lord Kelvin



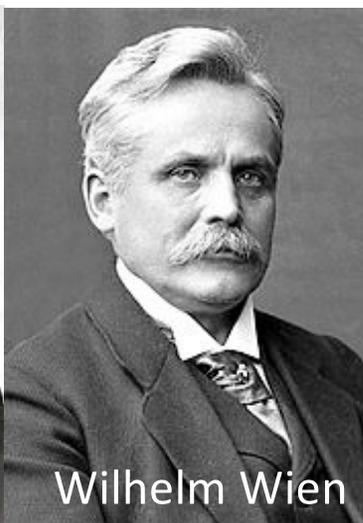
Ludwig Boltzmann



James C Maxwell

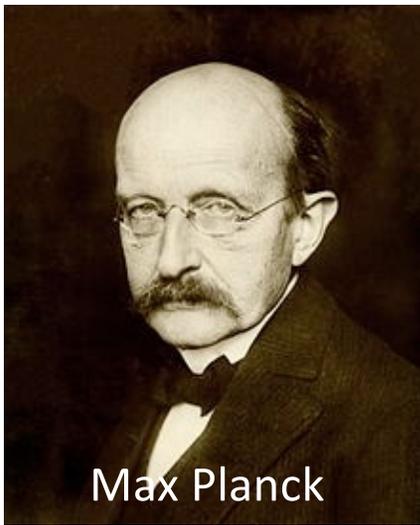


Johannes
van der Waals

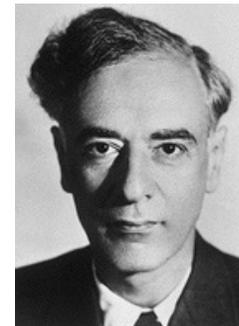
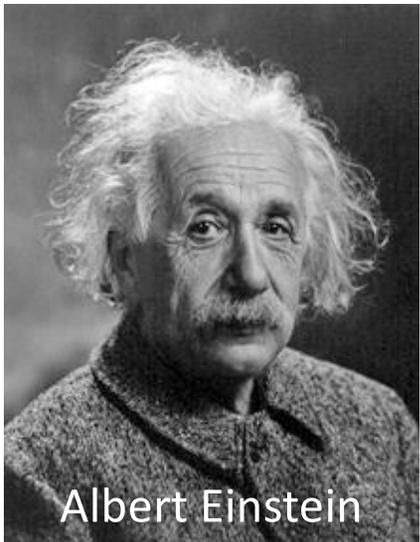


Wilhelm Wien

- **1850** *W. Thomson + H. v. Helmholtz*: 2nd law of thermodynamics
- **1857** *R. Clausius*: derivation of the equation of state for ideal gases
- **1860** *J.C. Maxwell*: Maxwell velocity distribution
- **1865** *R. Clausius*: “Entropy”, updated version of the 2nd law
- **1868–71** *L. Boltzmann*: generalization of the Maxwell distribution
- **1872** *L. Boltzmann*: H–theorem
- **1873** *van der Waals*: equation of state for real gases
- **1876** *L. Boltzmann*: transport equation
- **1876** *J.W. Gibbs*: thermodynamic potentials
- **1877** *L. Boltzmann*: $S = k \ln W$ (in words): statistical interpretation of entropy
- **1876/96/1909** *Loschmidt, Zermelo, Mach, Ostwald*: criticize Boltzmann’s kinetic gas theory: reversible equations – Poincare recurrence theorem apparently inconsistent with irreversibility of macroscopic behavior.
- **1894** *W. Wien*: black body radiation



- **1900** *M. Planck*: radiation law
- **1904/11** *W. Nernst/M. Planck*: 3rd law of thermodynamics
- **1911** *Ehrenfest*: establish quantum statistics
- **1916/17** *Chapman/Enskog*: expansion of the Boltzmann equation
- **1924** *Bose/A. Einstein*: Bose-Einstein statistics
- **1925/26** *W. Pauli/E. Fermi*: Fermi-Dirac statistics
- **1931** *L. Onsager*: theory of irreversible processes
- **1937** *L. Landau*: theory of phase transitions, extension on superconductors with Ginzburg, 1950
- **1943** *Chandrasekhar, Fowler*: application of stochastic methods in physics and astronomy
- **1944** *L. Onsager*: exact solution of the 2D Ising model (simple model with magnetism and other cooperative phenomena).



Ginzburg and Landau, 1950

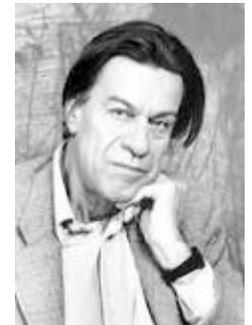
- **1956** *Bardeen, Cooper, Schrieffer*: explanation of superconductivity as Bose condensation
- **1956–58** *Landau*: theory of Fermi liquids
- since ~**1960** *Matsubara, Abrikosov, Gorkov, Dzyaloshinskii...*: application of quantum field theory in statistical physics
- **1966** *Kubo*: fluctuation-dissipation theorem
- since ~**1970**– *Haken, Prigogine...* structures out-of-equilibrium
Wilson, Fisher, Wegner,...: renormalization of strongly correlated systems
Flory, de Gennes... Application of statistical methods on polymers, liquid crystals ,... (or evaporation of black holes [Hawking radiation])
- since ~**1975** *Hopfield, Amit,...* theory of neural networks
- since ~**1985** *Bouchaud, ...*: application of methods of statistical physics on economic problems (“Phynance” or “Econo-Physics”)



Bardeen-Cooper-Schrieffer (1957)



Fisher, Wilson, Kadanoff



de Gennes

Topics

1. Introduction and background

- i. Role and tasks of Statistical Physics
- ii. Short history of Statistical Physics

done

2. Boltzmann's approach to Statistical Physics

- i. Classical mechanics, Liouville theorem
- ii. Micro- and macro-variables, thermal equilibrium
- iii. Boltzmann entropy
- iv. Quantum description
- v. Connection to thermodynamics, first and second law of thermodynamics

...

3. Gibbsian Ensemble

- i. Microscopic and macroscopic densities
- ii. Gibbs ensemble
- iii. Quantum description

4. Equilibrium Ensembles

- i. Microcanonical ensemble
- ii. Canonical ensemble
- iii. Grand canonical ensemble
- iv. Nernst theorem and third law of thermodynamics

...

5. Ideal Gases

- i. Classical ideal gases
- ii. Ideal quantum gases
- iii. Equation of state
- iv. Bose-Einstein condensation, superfluidity
- v. Photons
- vi. Fermions at low temperatures

6. Thermodynamics

- i. Thermodynamic potentials and thermodynamic stability
- ii. Response functions
- iii. Phase equilibrium
- iv. Van der Waals gas, Maxwell construction

7. Introduction to phase transitions

- i. Phase diagrams, phase transitions (1st/2nd order)
- ii. Critical phenomena (second order phase transitions)
- iii. Ginzburg-Landau theory

Literature

Recommended textbooks

K. Huang, *Statistical Mechanics*, John Wiley & Sons, New York (1987)

Frederick Reif, *Fundamentals of Statistical and Thermal Physics*, Waveland Pr Inc (2008)

R. Kubo, *Statistical Mechanics*, North Holland (1990)

Additional textbooks

L. Landau & I. Lifshitz, *Statistical Physics, Part 1: Volume 5*, Butterworth-Heinemann (1980)

F. Reif, *Fundamentals of statistical and thermal physics*, McGraw-Hill Book Company, New York (1965)

Sommerfeld, *Thermodynamics and Statistical Mechanics*, Academic press, New York (1956)

Advanced textbooks

P. Chaikin and T. Lubensky, *Principles of Condensed Matter Physics*, Cambridge University Press (1995)

R. P. Feynman, *Statistical Mechanics – A set of lectures*, Frontiers in Physics, Benjamin/Cummings, Reading Massachusetts (1982)

N. Goldenfeld, *Lectures on Phase transitions and the Renormalization Group*, Frontiers in Physics, Addison Wesley, Reading Massachusetts (1994)

L. Landau & I. Lifshitz, *Statistical Physics, Part 2: Volume 9*, Butterworth-Heinemann (1980)

lecture notes

- Summaries of the lectures will be available
- other sources:
 - Prof. M. Kardar: (MIT)
<https://ocw.mit.edu/courses/physics/8-333-statistical-mechanics-i-statistical-mechanics-of-particles-fall-2013/lecture-notes/>
 - Prof. Wei Cai (Stanford University)
micro.stanford.edu/~caiwei/me334/
 - Prof. Leonard Susskind (Stanford, youtube course)
www.youtube.com/playlist?list=PLB72416C707D85AB0

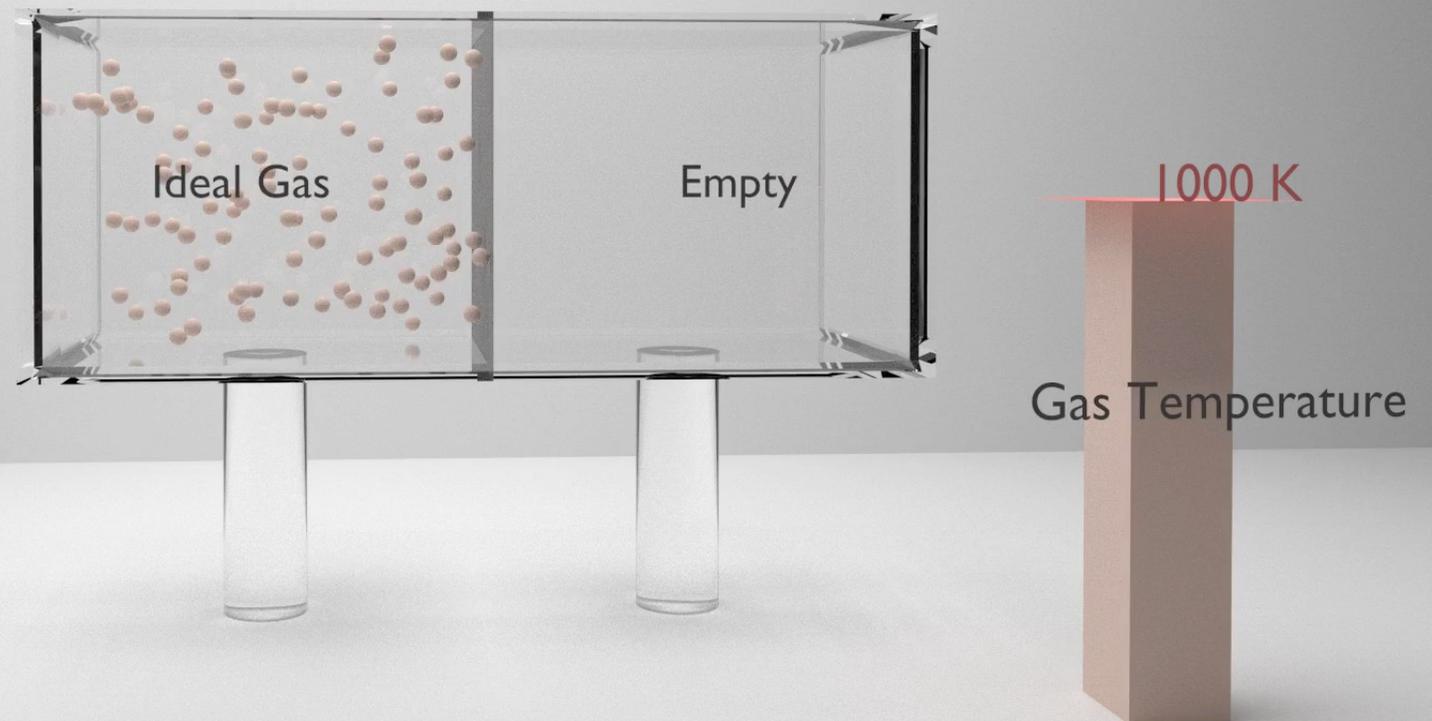
This Thursday:

Basics and Boltzmann's approach to Statistical Physics



$$S = k_B \ln(W)$$

Irreversible expansion of a gas



Irreversible process



(ensemble) volume in Γ -space

