

Computational Physics - PHYS 410/510

Spring 2023

Department of Physics - Northern Illinois University
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www.aglatz.net/teaching/compphys_S2023

Homework

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HW

due 2023-02-14, before class

Info

midterm task: **March 20-22, 2023**
final project presentation: **Tuesday, May 2, 2023, 9:30** (tentatively)
(will be assigned beginning of April.)

Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by * are for extra credit.

I. KEPLER PROBLEM - HAMILTON FORMALISM [5+10+20+10 PTS]

Write a program to solve numerically the Kepler problem. The Hamilton function of the problem in dimensionless form is defined as

$$H(\mathbf{q}, \mathbf{p}) = \frac{p_1^2 + p_2^2}{2} - \frac{1}{\sqrt{q_1^2 + q_2^2}}.$$

The initial conditions for the resulting equations of motion are defined by

$$p_1(0) = 0; \quad q_1(0) = 1 - e; \quad p_2(0) = \sqrt{\frac{1+e}{1-e}}; \quad q_2(0) = 0,$$

where e is the eccentricity, use 0.6 here to get elliptical orbits.

- Derive Hamilton's equations of motion. (do the calculations).
- Solve the equations of motion with above initial conditions with the explicit Euler method. (see book or lecture for useful parameter choices).
- The same as in b) but with the (explicit) symplectic Euler method. (see book or lecture)
- Modify the initial conditions and discuss the results! Try to confirm Kepler's laws of planetary motion with the help of your algorithm.

Plot the \mathbf{q} and \mathbf{p} trajectories, and the time dependent energy for b) and c). Hand in calculations/discussion and codes & plots.

II. ODE SOLVERS [25 PTS]

Solve the differential equation

$$\dot{y} = t^2 + y^2,$$

numerically with at least 4 different methods, but use the Taylor series and RK4 methods. Plot the solutions of all methods into one plot for a reasonable time interval. Do this for 2 or 3 different initial conditions of $y(0)$. Hand in codes & plots with descriptions.