

# Computational Physics - PHYS 410/510

## Spring 2022

Department of Physics - Northern Illinois University  
Prof. Andreas Glatz

www.aglatz.net/teaching/compphys\_S2023

### Homework

# 1

## HW

due 2023-01-31

#### Info

midterm task: **March 20-22, 2023**  
final project presentation: **Tuesday, May 2, 2023, 9:30** (tentatively)  
(will be assigned beginning of April.)

Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by \* are for extra credit.

#### I. COMPUTER ARITHMETIC [2+6+4+6+4 PTS]

- Give the standard (IEEE) single precision binary representation of the machine approximation for  $-1/3$ .
- Demonstrate how  $x_1 + x_2$  and  $x_1 - x_2$  are calculated on a computer using decimal floating point numbers with 4 significant digit precision for  $x_1 = 0.11258762 \cdot 10^2$  and  $x_2 = 0.11244891 \cdot 10^2$ . Calculate the relative error.
- Find smallest positive integer that is not exact in single precision.
- How many terms of the exponential series are needed to get the best single and double precision representation of  $e = \exp(1)$ ?
- Calculate the machine epsilon for single and double precision.

#### II. READ CHAPTER 2.4

#### III. NUMERICAL DIFFERENTIATION [5+10+15+10+5\* PTS]

Consider the finite interval  $I = [-a, a]$  with  $a = 5$  on the real axis. Define  $N + 1$  equally spaced grid-points  $x_i = -a + ih$ ,  $i = 0, \dots, N$  spanning  $I$  (i.e.,  $x_N = a$ ). Investigate the functions

$$g(x) = \exp(-x^2) \text{ and } h(x) = \sin(x),$$

on  $I$ .

- Define  $h$ . Plot these functions within the interval  $I$  by defining these functions on the grid-points  $x_i$  for a reasonable  $N$ .
- Plot the first derivative of these functions obtained analytically. (use same discretization and  $N$ )
- Calculate and plot the first derivatives of these functions by employing the first order backward, forward, and central difference derivatives. (Hand in code). Pay attention to the cases  $i = 0$  and  $i = N$ .
- Find the smallest value of  $N$  for each method and function, such that the relative error of numerical and analytical derivative is less than 1%.
- Calculate and plot the second derivative numerically using central differences (see chapter 2.4) for  $h(x)$  and find a suitable  $N$  such that the relative error is less than 1%.