Computational Physics - PHYS 410/510

Spring 2022

Department of Physics - Northern Illinois University
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Homework HW 5

due *2022-04-05*

Info	
	final project presentation: **Thursday, May 5, 2022**, *9:30-10:45* (will be assigned beginning of April.)
	Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by * are for extra credit.

I. POISSON EQUATION *[25+12+12+16 PTS]*

Here we solve the Poisson equation

$$\Delta\phi(x,y) = -\rho(x,y)$$

in two dimensions using the Gauss-Seidel iterative scheme given in lecture or book and the given convergence criterion. The boundary conditions shall be the Dirichlet conditions: $\phi = 0$ around the edge of the integration domain $[0, L_x] \times [0, L_y]$.

a) Implement the iterative Poisson solver for arbitrary charge density $\rho(x,y)$.

b) Reproduce the results for monopole, dipole, and quadropole using the parameters given in the book.

c) Solve for two separated oppositely charged monopoles (circular diameter 1, separated by e.g. $L_x/2$) or any other configuration you prefer. Adapt the system size and number of grid points appropriately.

d) Calculate the electric field $E(x,y)$ distribution by calculating the negative gradient of $\phi(x,y)$ using central differences for cases b) and c)

II. TIME-DEPENDENT HEAT EQUATION *[25 PTS]*

Implement a time-dependent heat equation solver and solve for fixed temperatures T_0 and T_N and initial $T_i = 0$ ($i \in [1, N-1]$) to evolve the temperature profile in time using the same Gaussian profile as for the stationary case. Use $L = 100$ and $N = 1000$ grid points. Other typical parameters are: $\kappa = 1$, $\Theta = -0.4$, $\ell = 1$, $T_0 = 0$, $T_N = 2.0$. Solve till you reach the stationary solution. Output the final temperature profile and four intermediate profiles. You can use the explicit Euler method, but make sure you fulfill the stability criterion for the time step size. The total number of time steps should be comparable to the ones used in example given in the lecture/book.