Computational Physics - PHYS 410/510
Spring 2020

Department of Physics - Northern Illinois University
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www.aglatz.net/teaching/compphys_S2020

Homework HW 3

due *2020-02-25*

Info	
	midterm exam: **Thursday, March 19, 2020**, *11:00-12:15* final project presentation: **Thursday, April 30, 2020**, *11:00* (will be assigned beginning of April.)
	Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by * are for extra credit.

I. DOUBLE PENDULUM *[5+10+20+15+10+15 PTS]*

a) Implement the double pendulum equations of motion using the RK4 scheme.

b) Reproduce/plot all 5 sets of ϕ_1-ϕ_2, p_1-p_2 and real space trajectories for the different initial conditions.

c) Produce Poincare plots by plotting (ϕ_1, p_1), whenever $\phi_2 = 0$ and $p_2 > 0$. The condition $\phi_2 = 0$ is substituted by $|\phi_2| < \epsilon$ in the numerical realization (choose an appropriate ϵ - explain your choice). Note that if the points are space filling the dynamics are chaotic, as discussed in the lecture. Try to find different initial conditions which result in regular behavior and different initial conditions which produce chaotic dynamics.

d) Let $x(t) = [\varphi_1(t), \varphi_2(t), p_1(t), p_2(t)]^T$ and $x'(t) = [\varphi_1'(t), \varphi_2'(t), p_1'(t), p_2'(t)]^T$ be two trajectories which correspond to different initial conditions x_0 and x_0'. In this case the distance between trajectories is defined as

$$d(t) = \sqrt{(\varphi_1(t) - \varphi_1'(t))^2 + (\varphi_2(t) - \varphi_2'(t))^2 + (p_1(t) - p_1'(t))^2 + (p_2(t) - p_2'(t))^2}$$

Plot the distance $d(t)$ as a function of time t for two different initial conditions.

e) Derive the double pendulum equations for $m_1 \neq m_2$ and $\ell_1 \neq \ell_2$ analytically.

f) Implement the Eqs. from e). What happens? What is the influence of ℓ_1 and ℓ_2 on the dynamics.