Computational Physics - PHYS 410/510
Spring 2020

Department of Physics - Northern Illinois University
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www.aglatz.net/teaching/compphys_S2020

Homework — HW 2

due *2020-02-13*

Info

midterm exam:	**Thursday, March 19, 2020**, *11:00-12:15*
final project presentation:	**Thursday, April 30, 2020**, *11:00* (will be assigned beginning of April.)

Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by * are for extra credit.

I. KEPLER PROBLEM - HAMILTON FORMALISM *[5+10+20+10 PTS]*

Write a program to solve numerically the Kepler problem. The Hamilton function of the problem in dimensionless form is defined as

$$H(\mathbf{q},\mathbf{p}) = \frac{p_1^2+p_2^2}{2} - \frac{1}{\sqrt{q_1^2+q_2^2}}\,.$$

The initial conditions for the resulting equations of motion are defined by

$$p_1(0)=0\,;\;\; q_1(0)=1-e\,;\;\; p_2(0)=\sqrt{\frac{1+e}{1-e}}\,;\;\; q_2(0)=0\,,$$

where e is the eccentricity, use 0.6 here to get elliptical orbits.

a) Derive Hamilton's equations of motion. (do the calculations).

b) Solve the equations of motion with above initial conditions with the explicit Euler method. (see book or lecture for useful parameter choices).

c) The same as in b) but with the (explicit) symplectic Euler method. (see book or lecture)

d) Modify the initial conditions and discuss the results! Try to confirm Kepler's laws of planetary motion with the help of your algorithm.

Plot the $\mathbf{q}$ and $\mathbf{p}$ trajectories, and the time dependent energy for b) and c). Hand in calculations/discussion and codes & plots.

II. ODE SOLVERS *[25 PTS]*

Solve the differential equation

$$\dot{y} = t^2 + y^2\,,$$

numerically with at least 4 different methods, but use the Taylor series and RK4 methods. Plot the solutions of all methods into one plot for a reasonable time interval. Do this for 2 or 3 different initial conditions of $y(0)$. Hand in codes & plots with descriptions.