Computational Physics - PHYS 410/510

Spring 2020

Department of Physics - Northern Illinois University
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Homework

HW 1

due *2020-01-28*

Info	
	midterm exam: **Thursday, March 19, 2020**, *11:00-12:15* final project presentation: **Thursday, April 30, 2020**, *11:00* (will be assigned beginning of April.)
	Program codes should be mailed to: aglatz@niu.edu (see also website). Other problem solutions can be handed in or mailed as well. Problems with points marked by * are for extra credit.

I. COMPUTER ARITHMETIC *[2+6+4+6+4 PTS]*

a) Give the standard (IEEE) single precision binary representation of the machine approximation for $-1/3$.

b) Demonstrate how $x_1 + x_2$ and $x_1 - x_2$ are calculated on a computer using decimal floating point numbers with 4 significant digit precision for $x_1 = 0.11258762 \cdot 10^2$ and $x_2 = 0.11244891 \cdot 10^2$. Calculate the relative error.

c) Find smallest positive integer that is not exact in single precision.

d) How many terms of the exponential series are needed to get the best single and double precision representation of $e = \exp(1)$?

e) Calculate the machine epsilon for single and double precision.

II. READ CHAPTER 2.4

III. NUMERICAL DIFFERENTIATION *[5+10+15+10+5* PTS]*

Consider the finite interval $I = [-a, a]$ with $a = 5$ on the real axis. Define $N + 1$ equally spaced grid-points $x_i = -a + ih$, $i = 0, \ldots, N$ spanning I (i.e., $x_N = a$). Investigate the functions

$$g(x) = \exp(-x^2) \text{ and } h(x) = \sin(x)\,,$$

on I.

a) Define h. Plot these functions within the interval I by defining these functions on the grid-points x_i for a reasonable N.

b) Plot the first derivative of these functions obtained analytically. (use same discretization and N)

c) Calculate and plot the first derivatives of these functions by employing the first order backward, forward, and central difference derivatives. (Hand in code). Pay attention to the cases $i = 0$ and $i = N$.

d) Find the smallest value of N for each method and function, such that the relative error of numerical and analytical derivative is less than 1%.

e) Calculate and plot the second derivative numerically using central differences (see chapter 2.4) for $h(x)$ and find a suitable N such that the relative error is less than 1%.